CERN-TH/2001-124

# PROBE OF SUSY AND EXTRA DIMENSIONS BY THE BROOKHAVEN g-2 EXPERIMENT

Pran Nath

*Theoretical Physics Division, CERN CH 1211, Geneva, Switzerland*

*Department of Physics, Northeastern University, Boston, MA 02115-5000, USA**



## Abstract

A brief review is given of $a_\mu = (g_\mu - 2)/2$ as a probe of supersymmetry and of extra dimensions. It is known since the early to mid nineteen eightees that the supersymmetric electro-weak correction to $a_\mu$ can be as large or larger than the Standard Model electro-weak correction and thus any experiment that proposes to test the Standard Model electro-weak correction will also test the supersymmetric correction and constrain supersymmetric models. The new physics effect seen in the Brookhaven (BNL) experiment is consistent with these early expectations. Detailed analyses within the well motivated supergravity unified model show that the size of the observed difference $(a_\mu^{exp} - a_\mu^{SM})$ seen at Brookhaven implies upper limits on sparticle masses in a mass range accessible to the direct observation of these particles at the Large Hadron Collider. Further, analyses also show that the BNL data is favorable for the detection of supersymmeteric dark matter in direct dark matter searches. The effect of large extra dimensions on $a_\mu$ is also discussed. It is shown that with the current limits on the size of extra dimensions, which imply that the inverse size of such dimensions lies in the TeV region, their effects on $a_\mu$ relative to

*Permanent address

the supersymmetric contribution is small and thus extra dimensions do not produce a serious background to the supersymmetric contribution. It is concluded that the analysis of the additional data currently underway at Brookhaven as well as a reduction of the hadronic error will help pin down the scale of weak scale supersymmetry even more precisely.

## 1. Introduction

The topics we discuss in this paper consist of the supersymmetric electro-weak effect on $a_\mu$, and the implications of the precise Brookhaven (BNL) $a_\mu$ result for the direct detection of supersymmetry at accelerators and in the direct search for dark matter. We will also discuss the effect of extra dimensions on $a_\mu$. We begin by reviewing the situation with regard to the Standard Model contribution which consists of $a_\mu^{SM} = a_\mu^{qed}+a_\mu^{had}+a_\mu^{EW}$. The qed corrections have been computed to $O(\alpha^5)$ (for a review see Refs.[1]), the Standard Model electro-weak correction is computed to one loop[2] and two loop orders[3] and is

$$a_\mu^{EW}(SM) = 15.1(0.4) \times 10^{-10} \tag{1}$$

The hadronic contribution from vacuum polarization corrections to $O(\alpha^2)$[4] and $O(\alpha^3)$[5] and light by light scattering contribution[6,7] together give[4,8] $a_\mu^{had} = 673.9(6.7) \times 10^{-10}$. The total Standard Model contribution is then $a_\mu^{SM}$=11659159.7 (6.7) $\times 10^{-10}$. The recent Brookhaven result gives a $2.6\sigma$ difference between experiment and theory[9]

$$a_\mu^{exp} - a_\mu^{SM} = 43(16) \times 10^{-10} \tag{2}$$

## 2. Supersymmetric electro-weak contributions

It is well known that the anomalous magnetic moment vanishes in the limit of exact supersymmetry[10]. The early analyses of $a_\mu$ in supersymmetric models with broken supersymmetry are listed in Ref.[11]. However, the anomalous moment is very sensitive to the

pattern of supersymmetry breaking and thus one needs phenomenologically viable models of SUSY breaking for such computations. The supergravity unified model[12,13] provides such a framework and led to the modern analyses of supersymmetric electro-weak correction to $a_\mu$[14,15]. The parameter space of the minimal supergravity model (mSUGRA) at low energy is characterized by the parameters $m_0, m_{\frac{1}{2}}, A_0, \tan\beta$ and sign$\mu$ where $m_0$ is the universal scalar mass, $m_{\frac{1}{2}}$ is the universal gaugino mass, $A_0$ is the universal trilinear coupling, $\tan\beta =< H_1 > / < H_2 >$ where, $H_2$ gives mass to the up quarks and $H_1$ gives mass to the down quarks and leptons, and $\mu$ is the Higgs mixing parameter which appears in the superpotential as $\mu H_1 H_2$. We reproduce here some of the results of Ref.[15] where the first full one loop analysis of the supersymmetric effect was given. The supersymmetric contribution here arises from the chargino - sneutrino exchange and from the neutralino - smuon exchange so that $a_\mu^{SUSY} = a_\mu^{\tilde{W}} + a_\mu^{\tilde{Z}}$. The chargino exchange which is typically the larger contribution is given by[17]

$$a_\mu^{\tilde{W}} = \frac{m_\mu^2}{48\pi^2}\frac{{A_R^{(a)}}^2}{m_{\tilde{W}_a}^2}F_1(\left(\frac{m_{\tilde{\nu}}}{m_{\tilde{W}_a}}\right)^2) + \frac{m_\mu}{8\pi^2}\frac{A_R^{(a)}A_L^{(a)}}{m_{\tilde{W}_a}}F_2(\left(\frac{m_{\tilde{\nu}}}{m_{\tilde{W}_a}}\right)^2) \tag{3}$$

where $A_L(A_R)$ are the left(right) chiral amplitudes and are given by[17]

$$A_R^{(1)} = -\frac{e}{\sqrt{2}\sin\theta_W}\cos\gamma_1; \quad A_L^{(1)} = (-1)^\theta \frac{e m_\mu \cos\gamma_2}{2M_W \sin\theta_W \cos\beta} \tag{4}$$

$$A_R^{(2)} = -\frac{e}{\sqrt{2}\sin\theta_W}\sin\gamma_1; \quad A_L^{(2)} = -\frac{e m_\mu \sin\gamma_2}{2M_W \sin\theta_W \cos\beta} \tag{5}$$

Here $\gamma_1$ and $\gamma_2$ are the mixing angles and $\theta = 0 (\theta = 1)$ if the lighter eigenvalue of the chargino mass matrix is negative (positive). The supersymmetric electro-weak corrections has several interesting features. First under the constraint of radiative breaking of the electro-weak symmetry one finds that the term proportional to $A_L A_R$ term in Eq.(3) dominates, and further this term itself is dominated by the light chargino exchange contribution. As is evident from Eq.(4) the light chargino exchange term carries with it a signature and because of that one finds a strong correlation between the sign of $\mu$ and the sign of $a_\mu^{SUSY}$. Thus one finds that quite generally that $a_\mu^{SUSY} > 0, \mu > 0$, and $a_\mu^{SUSY} < 0, \mu < 0$ (in the standard sign

convention[18]) except when $\tan\beta \sim 1$[16,17]. Second we also note that the dominant $A_L A_R$ term in Eq.(3) has a coupling proportional to $\sim 1/\cos\beta$. Because of this $a_\mu$ increases linearly with $\tan\beta$ for large $\tan\beta$[16,17]. As will be discussed in Sec.3 the effects of extra dimensions on $a_\mu$ is relatively small[19], so their contribution to $a_\mu$ does not pose a serious background to the supersymmetric electro-weak contribution. The effect of the phases was analyzed in Ref.[20] and it is found that the supersymmetric contribution to $a_\mu$ is a very sensitive function of the phases and their inclusion in the analysis can change both the sign and the magnitude of the supersymmetric contribution.

After the recent BNL experimental result became available[9] a large number of investigations exploring the implications of the result for supersymmetry have been reported[21–26]. In the analysis of Ref.[21] under the assumption of CP conservation and setting $a_\mu^{SUSY} = a_\mu^{exp} - a_\mu^{SM}$, it is found as anticipated[16,17] that the BNL data determines the sign of $\mu$ and one finds[21] $sign(\mu) = +1$. Further, using the $2\sigma$ error corridor of Eq.(2) one finds that the data implies upper bounds on the sparticle masses. Thus within mSUGRA one finds[21] $m_{\tilde{W}} \leq 650GeV, m_{\tilde{\nu}} \leq 1.5TeV$ and $m_{1/2} \leq 800GeV, m_0 \leq 1.5TeV$, for $\tan\beta \leq 55$. These results imply that the sparticles should become visible at the LHC and perhaps even at RUNII of the Tevatron. Additionally one finds that the $\mu$ sign implied by the BNL data is the sign which least restricts the supersymmetry parameter space under the $b \to s + \gamma$ constraint[27] and is also the one preferred in dark matter analyses. Thus the determination by the Brookhaven data that the sign of $\mu$ is positive is encouraging from the point of view of search for supersymmetric dark matter[21]. Further, as discussed above the CP violating phases associated with soft SUSY parameters can generate large contributions to $a_\mu$ and affect both the magnitude and the sign of $a_\mu$[20]. The above sensitivity of $a_\mu$ implies that the BNL data, i.e., Eq.(2), can provide a strong constraint on the phases. This indeed turns out to be the case and one finds that as much as sixty to ninety percent of the parameter space of the CP violating phases can be eliminated by the BNL data[25].

## 3. Effect of large extra dimensions on $a_\mu$

Next we discuss the implications of large extra dimensions[28] on $a_\mu$(for a recent review see Ref.[29]). This class of models can arise from compactifications of Type I string theory and in models of this type the string scale and even the fundamental Planck scale can be quite low, i.e., in the vicinity of a few TeV[28]. For specificity we shall consider the case with one large extra dimension compactified on $S^1/Z_2$ with compactification radius R where we assume that the inverse radius $M_R = 1/R$ is $O(TeV)$. In this model the resulting spectrum after compactification contains massless modes with N=1 supersymmetry in 4D, which precisely form the spectrum of MSSM in 4D and the massive Kaluza-Klein (KK) modes form N=2 multiplets in 4D. The Kaluza-Klein excitations generate corrections to the Fermi constant[30] and one finds for the above model $G_F = G_F^{SM}(1+\frac{\pi^2}{3}\frac{M_W^2}{M_R^2})$. With the current error corridor on $G_F^{SM}$ one finds $M_R \geq 3$TeV. Large extra dimensions affect the value of $a_\mu$ from contributions via the excitations of $W, Z, \gamma$ and the KK correction to $a_\mu$ is given by[19]

$$(\Delta a_\mu)^{extra} = \alpha\frac{\pi}{9}\frac{m_\mu^2}{M_R^2} + \frac{G_F m_\mu^2}{6\sqrt{2}}(-\frac{5}{12} + \frac{4}{3}(sin^2\theta_W - \frac{1}{4})^2)(\frac{M_Z^2 - M_W^2}{M_R^2}) \quad (6)$$

The relative minus sign between the $M_Z^2$ and the $M_W^2$ terms in the last brace in Eq.(6) arises because the Fermi constant must be normalized to take account of the KK correction. Numerically the effects of the KK states is small, i.e.,

$$a_\mu^{extra}/a_\mu^{SUSY} \leq O(10^{-2}) \quad (7)$$

Thus extra dimensions do not create a serious background to the SUSY contribution. The effect of extra dimensions could be enhanced in some models which, however, do not appear to be very natural[31]. Similar results are expected in models with strong gravity[32] since the fundamental Planck scale $M_*$ from the recent gravity experiment[33] is constrained so that $M_* \geq 3.5$TeV and this scale may be as high as 50-100 TeV from studies of graviton emission into large extra compact dimensions from a hot supernova core using the SN1987A data[34]. We note, however, that although the extra dimensions are invisible in g-2 experiment their

effects could still become visible at accelerators with large enough energies[29,35,36]. Additional analyses within the framework of extra dimensions can be found in Ref.[37].

## 4. Conclusion

SUSY provides the most natural explanation of the difference $a_\mu^{exp} - a_\mu^{SM}$ seen at BNL. The effect was already predicted within the framework of SUGRA models where it was known since the early to mid nineteen eightees that the supersymmetric correction could be as large or larger than the Standard Model electro-weak correction[15]. A detailed analysis of the implications of the BNL experiment in mSUGRA shows that if the size of the new physics effect seen by Brookhaven persists it would imply the existence of sparticles accessible at the LHC. Further, if SUSY is the right explanation of the difference seen in the BNL experiment, then the existence of a Higgs field as a fundamental field (as opposed to a composite field) is implied. In SUGRA unified models there is an upper limit of about 130 GeV for the lightest neutral Higgs boson within the usual naturalness limits on sparticle masses[38] and thus one expects this Higgs boson to become visible at RUNII of the Tevatron with appropriate integrated luminosity. Additionally the possibility of finding a sparticle at RUNII is not excluded. It is also found that the BNL data imposes impressive constraints on the phases of soft SUSY breaking parameters eliminating a large part of the parameter space of these phases. The effect of extra space-time dimensions on $a_\mu$ was also discussed and it is concluded that extra dimensions do not generate an effect comparable to the supersymmetric electro-weak effect. There are other possibilities not discussed here such as of a light higgs[39], lepto-quarks, composite models, techni-color and extra gauge bosons as possible sources for a large correction to $a_\mu$ (for a general review of these see Czarnecki and Marciano in Ref.[26] and for a model independent analysis see Ref.[40]). However, of all the scenarios mentioned above the possibility that the observed effect is arising from supersymmetry appears to us to be the most compelling.

In the coming months additional data collected in runs in the year 2000 will be analyzed

and one expects that the experimental error will reduce further. The central issue of course is the size of the difference $a_\mu^{exp} - a_\mu^{SM}$ and the associated error corridor. Here the crucial question concerns the size of the hadronic correction and the error associated with it. Since much of the error arises in the energy domain of less than 2 GeV in the $e^+e^- \rightarrow hadrons$ cross section more accurate data in this region will certainly help reduce the errors[8]. Another sensitive issue is the light by light contribution to the hadronic error. Although a computation of this correction relies entirely on theoretical models, it is comforting that two independent analyses[6,7] are in agreement on the overall sign and also in fair agreement on the magnitude of this contribution. These issues are expected to be explored in greater depth in the coming months and along with the expected more accurate experimental measurement of $a_\mu$ at Brookhaven, the revised version of Eq.(2) will pin down the scale of weak scale supersymmetry even more precisely.

## 5. Acknowledgements

The work reported here is a review of the collaborative work with Richard Arnowitt, Ali Chamseddine, and T.C. Yuan in the early nineteen eightees when the modern analysis of g-2 was carried out, of the more recent works with Utpal Chattopadhyay and Tarek Ibrahim, and of the work with Masahiro Yamaguchi on the effect of large extra dimensions on g-2. This research was supported in part by the NSF grant PHY-9901057.

## REFERENCES


1. A. Czarnecki and W.J. Marciano, Nucl. Phys. (Proc. Suppl.) **B76**, 245(1999).

2. K. Fujikawa, B. W. Lee, and A. I. Sanda, Phys. Rev. **D6**, 2923 (1972); R. Jackiw and S. Weinberg, Phys. Rev. **D5**, 2473 (1972); G. Altarelli, N. Cabbibo, and L. Maiani, Phys. Lett. **B40**, 415 (1972); I. Bars and M. Yoshimura, Phys. Rev. **D6**, 374 (1972); W. A. Bardeen, R. Gastmans, and B. E. Lautrup, Nucl. Phys. **B46**, 315 (1972).

3. A. Czarnecki and W.J. Marciano, Phys.Rev. Lett. **76**, 3267(1996); Phys. Rev. **D52**, R2619(1995); S. Peris, M.Perrottet, and E. de Rafael, Phys. Lett. **B355**, 523(1995); G. Degrassie and G.F. Giudice, Phys. Rev. **D58**, 053007(1998).

4. M. Davier and A. Hocker, Phys. Lett. **B435**, 427(1998).

5. B. Krause, Phys. Lett. **B390**, 392(1997).

6. J. Bijnens, E. Pallante, and J. Prades, Nucl. Phys. **B474**, 379(1996).

7. M. Hayakawa and T. Kinoshita, Phys. Rev. **D57**, 465(1998).

8. For an assessment of the current status of the hadronic contribution see, W.J. Marciano and B.L. Roberts, "Status of the hadronic contribution to the muon g-2 value", hep-ph/01015056.

9. H.N. Brown et al., Muon (g-2) Collaboration, hep-ex/0102017.

10. S. Ferrara and E.Remiddi, Phys. Lett. **53B**, 347(1974).

11. P. Fayet:In:Unification of the fundamental particle interactions, p. 587 eds. S. Ferrara, J. Ellis, P. van Nieuwenhuizen. New York:Plenum, 1980; J.A. Grifols, A. Mendez, Physical Review **D26** (1982) 1809; J. Ellis, J.S. Hagelin, D.V. Nanopoulos, Phys. Lett. **116B** (1982) 283; R. Barbieri, L. Maiani, Phys. Lett. **117B** (1982) 203.

12. A.H. Chamseddine, R. Arnowitt and P. Nath, Phys. Rev. Lett. **49**, 970(1982); R. Barbieri, S. Ferrara and C.A. Savoy, Phys. Lett. **B119**, 343(1982); L. Hall, J. Lykken, and S.

Weinberg, Phys. Rev. **D27**, 2359(1983): P. Nath, R. Arnowitt and A.H. Chamseddine, Nucl. Phys. **B227**, 121(1983).

13. P. Nath, R. Arnowitt and A.H. Chamseddine, "Applied N=1 Supergravity" (World Scientific, Singapore, 1984); H.P. Nilles, Phys. Rep. **110**, 1(1984); H. Haber and G.L. Kane, Phys. Rep. **117**, 75(1985); R. Arnowitt and P. Nath, in "Perspectives on Supersymmetry", World Scientific, ed. G. Kane. p.442(1998), hep-ph/9708254.

14. D.A. Kosower, L.M. Krauss, N. Sakai, Phys. Lett.**133B**, 305(1983).

15. T. C. Yuan, R. Arnowitt, A. H. Chamseddine and P. Nath, Z. Phys. **C26**, 407 (1984).

16. J.L. Lopez, D.V. Nanopoulos, X. Wang, Phys. Rev. **D49** (1994) 366.

17. U. Chattopadhyay and P. Nath, **D53**, 16481996); T. Moroi, Phys. Rev.**D53**, 6565(1996); M. Carena, M. Giudice and C.E.M. Wagner, Phys. Lett. **B390**, 234(1997); E. Gabrielli and U. Sarid, Phys. Rev. Lett. **79**, 4752(1997); K.T. Mahanthappa and S. Oh, Phys. Rev.**D62**, 015012(2000); T. Blazek, hep-ph/99122460; U.Chattopadhyay , D. K. Ghosh and S. Roy, Phys. Rev. **D62**, 115001 (2000).

18. SUGRA Working Group Collaboration (S. Abel et. al.), hep-ph/0003154.

19. P. Nath and M. Yamaguchi, Phys. Rev. **D60**, 116006(1999).

20. T. Ibrahim and P. Nath, Phys. Rev. **D61**, 095008(2000); T. Ibrahim and P. Nath, Phys. Rev. **D62**, 015004(2000); hep-ph/9908443.

21. U. Chattopadhyay and P. Nath, hep-ph/0102157.

22. L. Everett, G.L. Kane, S. Rigolin and L-T. Wang, hep-ph/0102145

23. J. Feng and K.T. Matchev, hep-ph/0102146

24. E.A. Baltz and P. Gondolo, hep-ph/0102147

25. T. Ibrahim, U. Chattopadhyay and P. Nath, hep-ph/0102324.

26. Additionally there is now a considerable body of work exploring the implications of the BNL experiment for supersymmetry. A partial list consists of

A. Czarnecki and W.J. Marciano, hep-ph/0102122; S. Komine, T. Moroi, and M. Yamaguchi, hep-ph/0102204;hep-ph/0103182; J. Hisano and K. Tobe, hep-ph/01202315; J. Ellis, D.V. Nanopoulos, K. A. Olive, hep-ph/0102331; R. Arnowitt, B. Dutta, B. Hu, Y. Santoso, hep-ph/0102344; K. Choi, K. Hwang, S. K. Kang, K.Y. Lee, W. Y. Song, hep-ph/0103048; R. A. Diaz, R. Martinez, J. A. Rodriguez, hep-ph/0103050; J. E. Kim, B. Kyae, H. M. Lee, hep-ph/0103054; S. K. Kang, K. Y. Lee, hep-ph/0103064; S. P. Martin, James D. Wells, hep-ph/0103067; K. Cheung, C-H Chou, O.C.W. Kong, hep-ph/0103183; S.Baek, P. Ko, H. S. Lee, hep-ph/0103218; D.F. Carvalho, J. Ellis, M.E. Gomez, S. Lola, hep-ph/0103256; A. Bartl, T. Gajdosik, E. Lunghi, A. Masiero, W. Porod, H. Stremnitzer, O. Vives, hep-ph/0103324; Y.-L. Wu and Y.-F. Zhou, hep-ph/0104056; B. Baek. T. Goto, Y. Okada and K. Okumura, hep-ph/0104146; C.-H. Chen and C.Q. Geng, hep-ph/0104151; K. Enqvist, E. Gabrielli, K. Huitu, hep-ph/0104174; A. Arhrib and S. Baek, hep-ph/0104225; D.-G. Cerdeno, E. Gabrielli, S. Khalil, C. Munoz and E. Torrente-Lujan, hep-ph/0104242; Y.G.Kim and M.M. Nojiri, hep-ph/0104258; Z. Chacko and G.D. Kribs, hep-ph/0104317; T. Blazek and S.F. King, hep-ph/0105005.

27. P. Nath and R. Arnowitt, Phys. Lett. **B336**, 395(1994); Phys. Rev. Lett. **74**, 4592(1995); F. Borzumati, M. Drees and M. Nojiri, Phys.Rev.**D51**, 341(1995); H. Baer, M. Brhlik, D. Castano and X. Tata, Phys. Rev. **D58**, 015007(1998).

28. I. Antoniadis, Phys. Lett. **B246** (1990) 377; E. Witten, Nucl. Phys. B **471** (1996) 135; J. Lykken, Phys. Rev. **D54** (1996) 3693; I. Antoniadis and M. Quirós, Phys. Lett. **B392** (1997) 61; N. Arkani-Hamed, S. Dimopoulos and G. Dvali, Phys. Lett. **B429** (1998) 263.

29. P. Nath, "Physics from Extra Dimensions", hep-ph/0011177.

30. P. Nath and M. Yamaguchi, Phys. Rev. **D60**, 116004(1999). M. Masip and A. Pomarol, Phys. Rev. **D60**,096005; W.J. Marciano, Phys. Rev. **D60**, 093006(1999); R. Casalbouni, S. Decourtis, D. Dominici and R. Gatto, Phys. Lett. **B462**, 48(1999); C.D. Carone, Phys. Rev. **D61**, 015008(2000).

31. K. Agashe, N.G. Deshpande, G.H. Wu, hep-ph/0103235

32. M.L. Graesser, Phys. Rev. **D61**, 074019(2000).

33. C.D. Hoyle, U. Schmidt, B.R. Heckel, E.G. Adelberger, J.H. Gundlach, D.J. Kapner, H.E. Swanson, "Sub-millimeter tests of the gravitational inverse square law: A search for large extra dimension", hep-ph/00110414.

34. S. Cullen and M. Perlestein, Phys. Rev. Lett.**83**, 268(1999).

35. P. Nath, Y. Yamada, and M. Yamaguchi, Phys. Lett.**B466**, 100(1999); See also S. Shiu, R. Shrock, and S.-H. Tye, Phys. **B458**, 274(1999); I. Antoniadis, K. Benakli and M. Quiros, Phys. Lett. **B460**, 176(1999); T. G. Rizzo, Phys. Rev. **D61**, 055005(2000).

36. For a review of the possibility of observing the effects of low scale quantum gravity, see M. Peskin, hep-ph/0002041; K. Sridhar, hep-ph/0004053.

37. H. Davoudiasl, J.L. Hewett, and T.G. Rizzo, Phys. Lett. **B493**, 135(2000); S.C. Park and H.S. Song, hep-ph/0103072; C.S. Kim, J.D. Kim and J. Song, hep-ph/0103127; X. Calmet and A.Nernov, hep-ph/0104278.

38. K.L. Chan, U. Chattopadhyay and P. Nath, Phys. Rev. **D58**, 096004(1998).

39. A. Dedes and H.E. Haber, hep-ph/0105014.; M. Krawczyk, hep-ph/0103223.

40. M.B. Einhorn and J. Wudka, hep-ph/0103034; M. Raidal, hep-ph/0103224.